ДИС Кирсев 08.11

$y = f(x)$ D

$x_1 < x_2$ $f(x_1) < f(x_2)$ $f'(x) > 0$ $x \in D$

$x = g(y)$ — $g(f(x)) = x$ - обратната функция

$f(g(y)) = y$

Допускаме че и е дефинирана, т.е има производна, тогава нейната, обратната функция или $g'(f(x)) \cdot f'(x) = 1 \Rightarrow$

$$g'(f(x)) = \frac{1}{f'(x)} \text{ или } g'(y) = \frac{1}{f'(g(y))}$$

при $x = g(y)$

$g(y + \Delta y) = x + \Delta x \Rightarrow \Delta x = g(y + \Delta y) - g(y)$

$f(g(y+\Delta y)) = y + \Delta y \Rightarrow \frac{\frac{1}{\Delta y}}{\Delta x} = \frac{\Delta x}{\Delta y}$

$\sin \frac{\pi}{2} = 1$; $\arcsin 1 = \frac{\pi}{2}$

arc - аркус - обратна функция

ако $\Delta y \to 0 \Rightarrow \Delta x \to 0$

но $\frac{\frac{1}{\Delta y}}{\Delta x} \to \frac{1}{f'(y)} \Rightarrow \frac{\frac{1}{\Delta y}}{\Delta x} = \frac{g(y+\Delta y) - g(y)}{\Delta y}$ има

граница, стремяща се към $\frac{1}{f'(y_0)}$

$(\arcsin x)'$ – обратная на $\sin x$

$y = \arcsin \Rightarrow x = \sin y$

$$(\arcsin x)' = \frac{1}{\cos(\arcsin x)} = \frac{1}{\sqrt{1-\sin^2(\arcsin x)}}$$

$\sin(\arcsin x) = 1$

$\cos(\arccos x) = 1$

$$\Rightarrow \frac{1}{\sqrt{1-x^2}}$$

$$(\arccos x)' = \frac{1}{-\sin(\arccos x)} = -\frac{1}{\sqrt{1-\cos^2(\arccos x)}} = -\frac{1}{\sqrt{1-x^2}}$$

$(\arcsin x + \arccos x) = !\ C$ – константа

$$(\operatorname{arctg})' = \cos^2(\operatorname{arctg} x) \Rightarrow \frac{1}{1+x^2}$$

$$\cos^2 u = \frac{1}{1+u^2} \qquad \sin^2 u = -\frac{1}{1+u^2}$$

$$(\operatorname{arcctg} x)' = -\frac{1}{1+\sin^2(\,)} = -\frac{1}{1+x^2}$$

$\operatorname{arctg} x + \operatorname{arcctg} x = C$

$\lim\limits_{x \to \infty} \operatorname{arctg} x = \pi/2$

$\lim\limits_{x \to \infty} \operatorname{arcctg} x = \pi$

логарифм от x – $\ln x$ и предел $u \to 0$

$$\frac{\ln(x+h)-\ln x}{h} = \frac{1}{h}\ln\frac{x+h}{x} = \ln\left(1+\frac{h}{x}\right)^{1/h} = \frac{1}{x}\ln e \to 1$$

$$\boxed{\lim_{x\to 0}(1+x)^{1/x} = e}$$

$$(\ln x)' = \frac{1}{x} \qquad y = \ln x$$

$y = f(x)$; $g(y) = x$; $g'(y) = \frac{1}{f'(g(y))}$

$$e^x = \frac{1}{\frac{1}{y}} = e^x$$

сложна функция

$$a^x = e^{x\ln a} = e^{\ln a^x} = a^x$$

$$(a^x)' = e^{x\ln a}\cdot \ln a$$

$$ch\,x = \frac{e^x + e^{-x}}{2}$$

производна

$$(ch\,x)' = \left(\frac{e^x - e^{-x}}{2}\right) = sh\,x$$

синус косинус

$$(sh\,x)' = ch\,x$$

$$\left(\ln\left(1+\sqrt{1+x^2}\right)\right)' = \frac{x}{\sqrt{1+x^2}+1+x^2}$$

$$\left(\ln\left(x+\sqrt{1+x^2}\right)\right) = \frac{1}{\sqrt{1+x^2}}$$

Φ $x^\alpha = e^{\alpha\ln x}$ само за $x>0$

$$(x^\alpha)' = (e^{\alpha\ln x})' = e^{\alpha\ln x}\,\alpha\frac{1}{x} = \alpha x^{\alpha-1}$$

$$x^x = f(x) = e^{x \ln x}$$

$$(x^x)' = (f(x))' = e^{x \ln x} (\ln x + 1) = x^x (1 + \ln x)$$

Ако $f(x) = u(x)^{v(x)}$

$\frac{f'}{f}$ – логаритмична производна

$$\frac{f'}{f} = v' \ln u + \frac{v}{u} u' \Rightarrow f' = f \left(v' \ln u + \frac{v}{u} u' \right) =$$

$$= u^v \left(v' \ln u + \frac{v}{u} u' \right)$$